Change Point Analysis for Intelligent Agents in City Traffic

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Abstract. Change point (CP) detection is an important problem in data mining (DM) applications. We consider this problem solving in multi-agent systems (MAS) domains. Change point testing allows agents to recognize changes in the environment, to detect more accurately current state information and provide more appropriate information for decision-making. Standard statistical procedures for change point detection, based on maximum likelihood estimators, are complex and require construction of parametrical models of data. In methods of computational statistics, such as bootstraping or resampling, complex statistical inference is replaced by a large computation volumes. However, these methods require accurate analysis of their precision. In this paper, we apply and analyze a bootstrap-based CUSUM test for change point detection, as well as propose a pairwise resampling test. We derive some useful properties of the tests and demonstrate their application in the decentralized decision-making of vehicle agents in city traffic.

Keywords: Multiagent decision-making, data mining, change point detection, resampling, variance, bootstrapping CUSUM test, traffic control

1 Introduction

Change point (CP) analysis is an important problem in data mining (DM), the purpose of which is to determine if and when a change in a data set has occurred. In multiagent systems (MAS) research, methods of CP analysis are applied, but not widely. One of the most popular agent-related areas of CP analysis application is web mining. Here, agents deal with automatic knowledge discovery from web documents and services, including social networks. CP detection in values of different parameters, such as number of messages, frequency of certain actions, or number of active users in some blogs are relevant for a wide number of applications, such as marketing, production, security. For example, Lu et. al. [10] applied the CUSUM algorithm in combination with shared beliefs for agent-oriented detection of network attacks, McCulloh [9] for detection of changes in social networks.

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A traditional statistical approach to the problem of CP detection is maximum likelihood estimation (MLE) [5],[8]. In this approach, an appropriate data model is constructed, which includes a model of CP. Then a likelihood function for model parameters (including CP) is designed and an estimator of for CP is obtained as a result of the likelihood function minimization. Such an approach requires assumptions about the data model and underlying distributions as well as complex analytical or numerical manipulations with a likelihood function.

As an alternative to the classical approach, methods of computational statistics, like bootstrap or resampling, are widely used [7]. In such methods, complex analytical procedures are replaced by intensive computation, which is effective with modern computers. However, these methods only provide approximate solutions, and the analysis of their accuracy and convergence rate is very important for their correct application.

One of the most popular methods of computational statistics, used for CP analysis, is a cumulative sum bootstrapping test (CUSUM bootstrapping test) [4]. This method does not require assumptions about data models and corresponding distributions; the idea of the test is to construct so-called CUSUM plots: one on the initial sample and one on each of a large number on permuted (bootstrapped) samples. The difference between the initial plot and bootstrapped plots aids to spread regarding the CP existence. This test relies on a visual assessment of whether there is a change in the slope of the CUSUM plot (see Figure 3).

In previous publications, we applied the resampling method for analysis and DM in different kinds of systems, including information, reliability, transportation logistics, software systems, including MAS [1],[3].

The purpose of this paper is twofold. First, we explain how non-parametrical CP detection methods may be integrated into agent decision support by illustrating it on detecentralized traffic routing scenario. Second, we demonstrate how to analyze a precision of the considered tests, taking expectation and variance of resulting estimators as an efficiency criteria.

We discuss two CP tests based on methods of computational statistics: a bootstrap-based CUSUM test, and a novel test, called pairwise resampling test. We analyze the efficiency of both tests as well as show how these tests are applied in MAS systems, focusing on the traffic applications. Case studies are presented to demonstrate how CP analysis is incorporated into agents decision-making processes to verify the potential effect of the proposed approach.

The paper is organized as follows. In Section 2, we describe how CP analysis is incorporated in the decision module of agents. In Section 3, we formulate the CP problem and explain a standard CUSUM bootstrapping approach. In Section 4, we present proposed CP detection algorithms. In Section 5, we provide the most important aspects of the tests efficiency analysis. Section 6 demonstrates a case study, where the proposed methods are used in decentralized traffic scenario. Section 6 contains final remarks, conclusions and outlook.

2 CP Based Decision Making for Agents in Traffic

Appropriate DM tools are very important for a class of MAS where the individual agents use DM technologies to construct and improve a basis for local decision-making and use communication, coordination and cooperation mechanisms in order to improve local decision-making models and to provide a basis for joint decision-making [11]. In order to make appropriate decisions, agents analyze incoming data flows, construct relevant models and estimate their parameters.

The environment and behavior of other agents are subject to changes. Suppose, some other agent decides to change its plans and start acting in a new manner, some part of the environment may become unavailable for agents, new agents may appear etc. So, the old behavior of the agent becomes inappropriate.

Let us consider a simple example from a traffic domain. Let a vehicle agent plans its route through a street network of a city. The agent is equipped with a receiver device, which allows obtaining an information about times needed to travel through the streets. Based on this information, the vehicle agent makes strategic decisions regarding its route. The vehicle does not use only messages from TMC: rather it builds some model based on historical information (such a model is considered by Fiosins et. al. [6]). In the simplest case, the vehicle agent just calculates an average of some set of historical observations.

Now suppose that some change occurs (traffic accident, overloading of some street etc.). The purpose of the agent is to detect this change and make appropriate re-routing decisions.

Let us describe an architecture of an intelligent agent from this point of view. It receives observations (percepts) from the environment as well as communication from other agents (Fig. 1). Communication subsystem is responsible for receiving input data, which then is pre-processed by an initial data processing module. The information, necessary for decision-making, is obtained from initial data by constructing corresponding data models (regression, time series, rule-based etc.). A data models estimation/learning module is responsible for the estimation of the model parameters or iterative estimation ("learning"), which provides an information base for an agent. Mentioned blocks represent a DM module of the agent. The information then is transformed to an internal state of the agent, which is agent's model of the real world. It represents the agent knowledge about the environment/other agents. Based on the internal state, the agent performs its decision-making. The efficiency (utility) functions measure an accordance of the internal state as a model of the external (environment state) with goals of the agent. Based on it, the agent produces (updates) its plan to reach its goal; this process may include the internal state change. As well, the efficiency function itself (or their parameters) can change under learning process.

The decision-making process includes strategic decisions of the agents [6], which define plans of general resource distribution as well as tactical decisions, including operative decisions regarding resource usage. For example, strategic decisions of a vehicle agent may include the route choice, but tactical decisions may include include speed/lane selection. As well, the agent plans its social behavior, i.e. its interactions with other agents. The result of this process is a construction of a plan (policy). The plan is given to an action module for the actual action selection and execution.

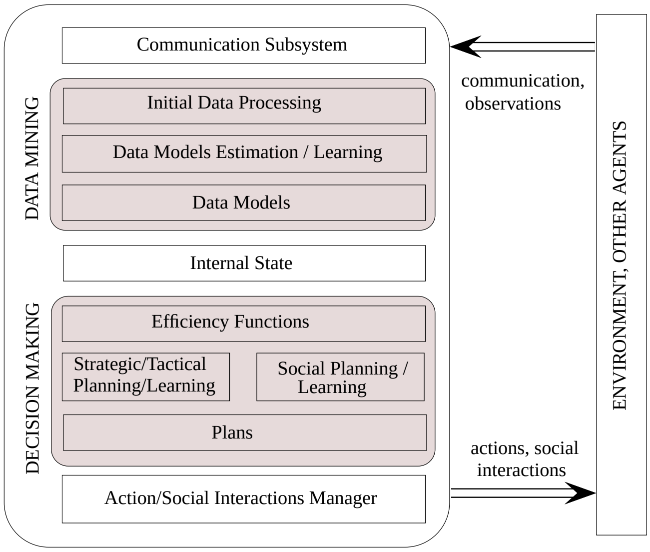

Fig. 1. An agent architecture including DM and decision making modules

Consider an example of DM module of an autonomous vehicle agent.

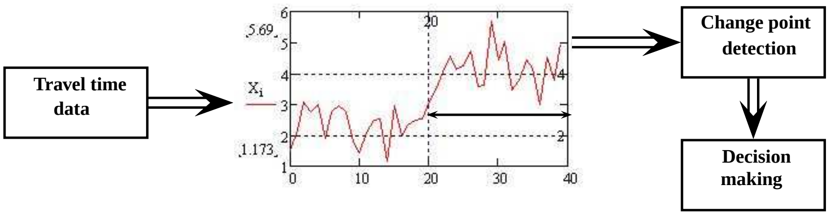

Fig. 2. DM module for strategic (routing) information processing

The vehicle receives travel time data, which are pre-processed by the initial data processing module. Then data is tested on the existence of CP. If it is detected, only data after the last CP are used in future analysis, where the current state (travel time) trough a given street is estimated. In the simplest case an average of travel times after the last CP is used.

In the next Section we present a CP detection problem as well as describe a standard bootstrap-based CUSUM test for CP detection.

3 CP Problem and CUSUM Test

Let us formulate a CP detection problem. Let $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ be a random sample. Let us divide this sample as $\mathbf{X} = \{\mathbf{X}^B, \mathbf{X}^A\}$, where $\mathbf{X}^B = \{x_1^B, x_2^B, \ldots, x_k^B\}$, $\mathbf{X}^A = \{x_1^A, x_2^A, \ldots, x_{n-k}^A\}$. We say that there is a CP at position k in this sample, if elements $\mathbf{X}^B$ are distributed according to a distribution function (cdf) $F_B(x)$, but elements $\mathbf{X}^A$ according to cdf $F_A(x) \neq F_B(x)$. The aim of a CP detection test is to estimate the value of k (clear that in the case of $k = n$ there is no CP). We are interested in a case when $F_B(x)$ and $F_A(x)$ differ by a mean value.

Note that a CP is not always visually detectable. Figure 3 represents two samples: left has exponential distribution, right has normal. Both have a CP at $k = 10$: for the exponential distribution its parameter λ changes from 1/10 to 1/20; for the normal distribution its parameter μ changes from 5 to 7. One should have an experience to see these CP visually.

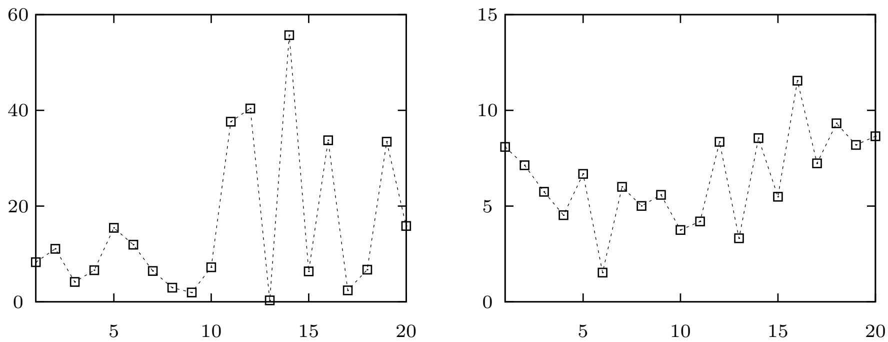

Fig. 3. A sample of 20 observations from exponential (left) and normal (right) distributions with CP at $k = 10$

A popular non-parametric approach for CP analysis is bootstrapping CUSUM test. CUSUM presents the cumulative sum of the differences between individual data values and the mean. If there is no shift in the mean of the data, the chart will be relatively flat with no pronounced changes in slope. Also, the range (the difference between the highest and lowest data points) will be small. A data set with a shift in the mean will have a slope change at the data point where the change occurred, and the range will be relatively large.

The cumulative sum S_i at each point i is calculated by the sample $\mathbf{X}$ by adding the difference between a current value and sample mean to the previous sum as

$$S_i = \sum_{j=1}^{i} (x_j - \bar{X}), \tag{1}$$

where $\bar{X}$ is the mean of the sample $\mathbf{X}$, $i = 1, 2 \ldots n$.

A CUSUM chart starting at zero will always end with zero as well: $S_n = 0$. If a CUSUM chart slopes down, it indicates that most of the data are below the mean. A change in the direction of a CUSUM indicates a shift in the average. At the CP, the slope changes direction and increases, indicating that most of the data points are now greater than the average.

In order to make a CP detection procedure more formal, a measure of the initial CUSUM line divergence from "normal" lines for given data is calculated. It is calculated using a technique known as bootstrapping, whereby N random permutations $\mathbf{X}^{*j}$, $j = 1, 2, \ldots, N$ of $\mathbf{X}$ are generated and corresponding CUSUMS S_i^{*j} are calculated by formula (1).

For a fixed point k the percentage of times where the cumulative sum for the original data exceeds the cumulative sum for the randomized bootstrap data is calculated as $p_k^* = \#\{j : S_k^{*j} \leq S_k\}/N$.

For values of p_k^* near 0 or near 1 we can say that the CP in k occurs. The idea behind this is that values S_k^{*j} approximate the distribution of CUSUMs constructed till k under assumption that data is mixed (values may be taken both before the CP and after the CP).

An example of a CUSUM test is presented on Figure 4.

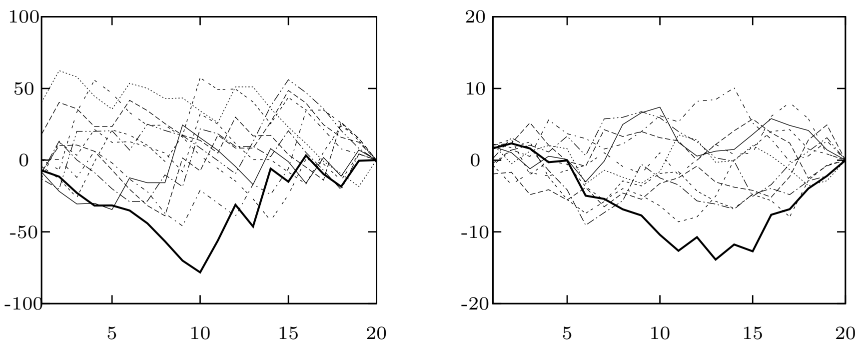

Fig. 4. CUSUM test examples with CP at $k = 10$

Here we can see a minimum of original CUSUM line at point $k = 10$; for other bootstrapped lines there is no minimum.

However, is this test reliable? Will the original CUSUM line be always outside of bootstrapped CUSUM line range? If not, then what is an expected value of the percentage of bootstrapped CUSUMS, which the original CUSUM exceeds? How this percentage differs from its expected value? All these questions should be answered during the method analysis phase; without the accurate analysis the method cannot be correctly applied.

4 Resampling Based CP Tests

4.1 CUSUM Based Test

Let us describe an algorithm for a CUSUM-based test for a CP at the point k. On r-th resampling step, we extract, without replacement, k elements from a sample $\mathbf{X}$ of size n, forming the resample $\mathbf{X}_k^{*r} = \{x_1^{*r}, x_2^{*r}, \dots, x_k^{*r}\}$. Then we construct r-th resampling CUSUM

$$S^{*r} = \sum_{i=1}^{k}(x_i^{*r} - \bar{X}) = \sum_{i=1}^{k} x_i^{*r} - k\bar{X}, \tag{2}$$

where $\bar{X}$ is an average over the initial sample $\mathbf{X}$.

Each CUSUM is compared with a pre-defined value x, obtaining an indicator value $\zeta^{*r} = 1_{S^{*r} \le x}$.

We make N such realizations, obtaining a sequence of indicators $\zeta^{*1}, \zeta^{*2}, \dots, \zeta^{*N}$. These values in fact approximate a cdf of CUSUMS. We estimate it as

$$F^*(x) = P\{S^{*r} \le x\} = \frac{1}{N}\sum_{r=1}^{N}\zeta^{*r}. \tag{3}$$

As a value of x we take a value of a CUSUM S, calculated by initial data. So the probability of interest is $F^*(S)$.

Low or high value of this probability allows to spread about CP existence. In principle, we can find maximal (close to 1) or minimal (close to 0) value of this probability on all k and consider this point as a CP.

A corresponding calculation of the probability $F^*(S)$ is presented in Algorithm 1.

Algorithm 1 Function CUSUM_TEST

function CUSUM_TEST($\mathbf{X}, k, N$)
    $S = \sum_{i=1}^{k}(x_i - \bar{X})$
    for $r = 1 \dots N$ **do**
        $\mathbf{X}_k^{*r} \leftarrow$ **resample**($\mathbf{X}, k$)
        $S^{*r} = \sum_{i=1}^{k}(x_i^{*r} - \bar{X})$
        if $S^{*r} < S$ **then** $\zeta^{*r} = 1$
        else $\zeta^{*r} = 0$
    end for
    return $1/N \sum_{r=1}^{N} \zeta^{*r}$
end function

However, as the same elements can be used for calculation of ζ^{*r} on different realizations r, it leads to a complex structure of dependency between ζ^{*r}. So we should be accurate in results interpretation here. In Section 5 we provide the most important aspects of this test analysis.

4.2 Pairwise Resampling CP Test

We propose an alternative resampling-based CP test; we call it pairwise resampling test. It is based on calculation of the probability $P\{Y \leq Z\}$ that one random variable (r.v.) Y is less than another r.v. Z [2]. Suppose that the sample $\mathbf{X}^B$ contains realizations of some r.v. Y, but the sample $\mathbf{X}^A$ realizations of some r.v. Z. Our characteristic of interest is the probability $\Theta = P\{Y \leq Z\}$.

On r-th resampling step we extract one value y^{*r} and z^{*r} from the samples $\mathbf{X}^B$ and $\mathbf{X}^A$ correspondingly and calculate an indicator value $\zeta^{*r} = 1_{y^{*r} \leq z^{*r}}$.

We make N such realizations, obtaining a sequence of indicators $\zeta^{*1}, \zeta^{*2}, \ldots, \zeta^{*N}$. The resampling estimator of Θ is

$$\Theta^* = \frac{1}{N} \sum_{r=1}^{N} \zeta^{*r}. \tag{4}$$

In order to check if there is a CP, we construct a confidence interval for Θ. We produce v such estimators, denote them $\Theta_1^*, \Theta_2^*, \ldots, \Theta_v^*$. Let us order them, producing an ordered sequence $\Theta_{(1)}^* \leq \Theta_{(2)}^* \leq \ldots \leq \Theta_{(v)}^*$.

Let us select a confidence probability γ for this interval (γ is usually selected 0.95 or 0.99). We accept $[\Theta^*_{(\lfloor \frac{1-\gamma}{2} v \rfloor)}; \Theta^*_{(\lfloor \frac{\gamma}{2} v \rfloor)}]$ as a γ confidence interval for Θ.

Note that in the case of CP absence the probability Θ will be equal to 0.5, and the estimators Θ^* will be close, but different from 0.5. However, is this difference significant? In order to answer we check if value 0.5 traps into the constructed confidence interval (Algorithm 2).

Algorithm 2 Function PAIRWISE_CONFIDENCE

function PAIRWISE_CONFIDENCE($\mathbf{X}, k, N, v, \gamma$)
  $\mathbf{X}^B =$ **subsample**$(\mathbf{X}, 1, k)$, $\mathbf{X}^A =$ **subsample**$(\mathbf{X}, k+1, n)$
  for $j = 1 \ldots v$ **do**
    for $r = 1 \ldots N$ **do**
      $y^{*r} \leftarrow$ **resample**$(\mathbf{X}^B, 1)$, $z^{*r} \leftarrow$ **resample**$(\mathbf{X}^A, 1)$
      if $y^{*r} < z^{*r}$ **then** $\zeta^{*r} = 1$
      else $\zeta^{*r} = 0$
    end for
    $\Theta_j^* = \sum_{r=1}^{N} \zeta^{*r}$
  end for
  sortΘ^*
  return $[\Theta^*_{(\lfloor \frac{1-\gamma}{2} v \rfloor)}; \Theta^*_{(\lfloor \frac{\gamma}{2} v \rfloor)}]$
end function

There is again a complex dependence structure between ζ^{*r}, and so between Θ^*, because the same elements may be used in comparisons on different realizations. So true coverage probability of constructed interval will differ from γ. The goal of the algorithm analysis is to calculate the true coverage probability of this interval; then we can correctly apply the method.

5 Analysis of the CP tests accuracy

In this Section, we shortly highlight the most important aspects of the methods efficiency analysis. The complete analysis can be found in our articles [1],[2],[3].

5.1 CUSUM Based Test

We are going to calculate an expectation and variance of the estimator (3). This means that we calculate theoretically an average of the estimator and spread of the percentage of cases, when the CUSUM constructed on the original data exceeds CUSUMS constructed on the bootstrapped data.

Let y_r be a number of elements, extracted from $\mathbf{X}^B$; then from $\mathbf{X}^A$ we extract $k - y_r$ elements. Then the expectation of (3) can be expressed as

$$E[F^*(x)] = P\{S^{*r} \leq x\} = \sum_{y_r=2}^{k-1} \int_{-\infty}^{\infty} F_B^{(y_r)}(x-u) dF_A^{(k-y_r)}(u) \cdot p_{y_r}(p), \qquad (5)$$

where $F^{(k)}(x)$ is a convolution of the cdf $F(x)$ with itself.

Variance of (3) can be expressed as

$$Var[F^*(x)] = \frac{1}{N} Var\left[1_{\{S^{*r} \leq x\}}\right] + \frac{(N-1)}{N} Cov\left[1_{\{S^{*r} \leq x\}}, 1_{\{S^{*p} \leq x\}}\right], \qquad (6)$$

for $r \neq p$.

Only the covariance term depends on the resampling procedure, which can be expressed using the mixed moment $\mu_{11} = \left[1_{\{S^{*r} \leq x\}} \cdot 1_{\{S^{*p} \leq x\}}\right]$.

In order to calculate μ_{11}, we use the notation of $\boldsymbol{\alpha}$-pair [1],[3]. Let $\boldsymbol{\alpha} = (\alpha_B, \alpha_A)$, where α_B and α_A are the number of common elements extracted from $\mathbf{X}^B$ and $\mathbf{X}^A$ correspondingly on two different resampling realizations.

Than μ_{11} can be expressed by fixing all possible values of $\boldsymbol{\alpha}$:

$$\mu_{11} = \sum_{\boldsymbol{\alpha}} \mu_{11}(\boldsymbol{\alpha}) P(\boldsymbol{\alpha}). \qquad (7)$$

For the case of exponential and normal distributions we can obtain explicit formulas for the previous expressions.

5.2 Pairwise Resampling CP Test

In order to analyze properties of (4), we introduce a protocol notation [2]. Let us order a sample $\mathbf{X}^B$, obtaining an ordered sequence $\{x^B_{(1)}, x^B_{(2)}, \ldots, x^B_{(k)}\}$. Let $c_i = \#\{x^A_j \in \mathbf{X}^A : x^B_{(i-1)} \leq x^A_j \leq x^B_{(i)}\}$, $x^B_{(0)} = -\infty$, $x^B_{(k+1)} = \infty$. We call $k+1$-dimensional vector $C = \{c_1, c_2, \ldots, c_{k+1}\}$ as a protocol.

For a fixed protocol C the conditional probability of the event $\{Y \leq Z\}$ is

$$q_C = P\{Y \leq Z | C\} = \frac{1}{k(n-k)} \sum_{i=1}^{k} \sum_{j=i}^{k} c_j. \qquad (8)$$

The probability that one resampling estimator Θ_j^* will be less than Θ is given by the binomial distribution with a probability of success (8):

$$\rho_C = P\{\Theta_j^* \leq \Theta | C\} = \sum_{\zeta=0}^{\Theta r-1} \binom{r}{\zeta} q_C^{\zeta} (1 - q_C)^{r-\zeta}. \quad (9)$$

Finally the unconditional probability of coverage is calculated as $\sum_C P_C \cdot R_C$.

6 Case Study

We consider a vehicle routing problem in a street network, where vehicles receive data about travel times and are applying the shortest path algorithm looking for a fastest path to their destination. As travel times are subject to change, that's why CP analysis is performed. If CP is detected, only the data part after the last CP is taken into account.

We suppose, that travel times trough the streets are normally distributed and are subject to changes in the mean. An example of input data is shown in Figure 5 (left). The vehicle analyses CPs in such data for all streets and selects an appropriate fastest route; the route selection process is presented in Figure 5 (right).

Now consider the behavior of CP estimators. In Figure 6 (left) the CUSUM test presented in the case of CP absence. In this case, we can see a big variance of the probability $F^*(x)$ of interest (standard deviation = 0.29). This means, that there exist a big risk of considering some point as a CP, if it is not one. Figure 6 (right) demonstrates the CUSUM test in the case of CP existence. Here we see very good CP detection with practically zero variance at the CP.

Now let us consider the pairwise test. In Figure 7 (left) we see this test in the case of CP absence. Here we see smaller variance of the probability Θ^* of interest (standard deviation = 0.14 on the most of the interval). This means, that a risk of considering some point as a CP, if it is not one, is lower than for the CUSUM test. Figure 7 (right) demonstrates this test in the case of CP. Here detection is not so bad as well, however the variance of the estimator is bigger, so there is a risk to miss this CP.

We can conclude that the CUSUM test detects CP very well; however, it may consider as CP some point, which is not one. In opposite, the pairwise test is more reliable in the case of a CP absence; however, it can miss some CPs.

So for streets where CPs are rare, it is better to use the pairwise test; the CUSUM test is better for streets with often occurred CPs in travel times.

7 Conclusions

CP detection is very important task of DM for MAS, because it allows agents to estimate more accurate the environment state and prepare more relevant information for decentralized planning and decision-making. As classical statistical methods for CP estimation are relatively complex, it is better to apply methods of computational statistics for this problem.

In this paper, we considered an application of CP detection as a part of DM module of an intelligent agent. We considered two resampling-based CP detection tests: CUSUM-based bootstrapping test and pairwise resampling test. We described algorithms of their application as well as highlighted the most important aspects of their efficiency analysis, taking expectation and variance of the estimators as the efficiency criteria.

We demonstrated an application of CP detection for vehicle agents in city traffic. This allows vehicles to detect CPs in street travel times and select more appropriate path in a street network.

The first test demonstrated good detection of CPs, however has a big variance in the case of CPs absence. The second test has smaller variance in this case, however worse detects existing CPs.

First experiments show that the demonstrated approach allows reducing the travel time of vehicles. In the future we will work on an application of computational statistics methods for different DM procedures in MAS. Another important direction is an application of our approach for different domains.

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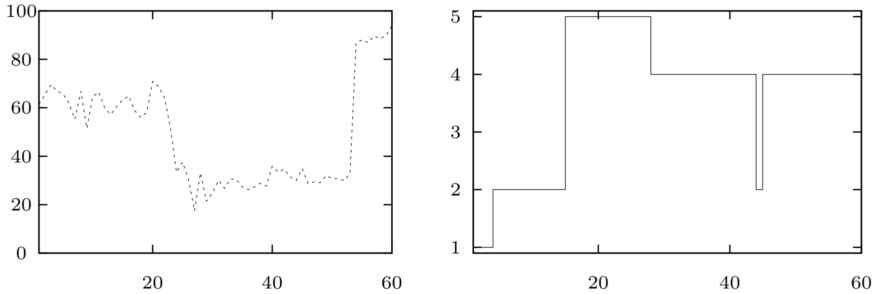

Fig. 5. Travel times on one street with 60 observations (left) and resulting route selection from 5 routes (right)

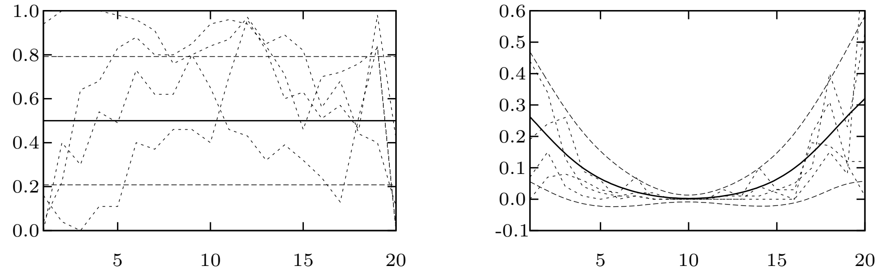

Fig. 6. Results of the CUSUM test without CP (left) and with CP at $k = 10$ (right) for a fragment of 20 observations. Straight line shows the expected value $E[F^*(x)]$ of the estimator $F^*(x)$, dashed lines show the difference between the expected value and standard deviation $E[F^*(x)] - Var[F^*(x)]^{1/2}$ of the estimator $F^*(x)$, dotted lines show several realizations of $F^*(x)$

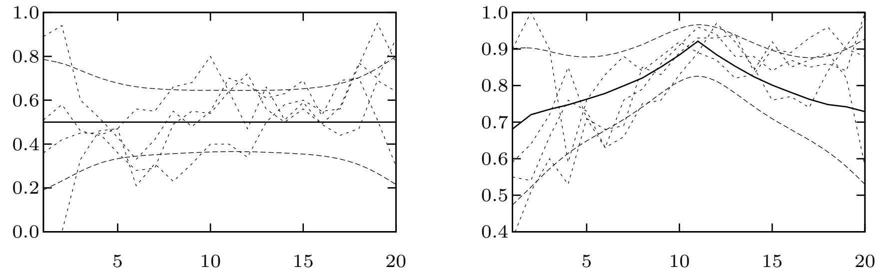

Fig. 7. Results of the pairwise test without CP (left) and with CP at $k = 10$ (right) for a fragment of 20 observations. Straight line shows the expected value $E[\Theta^*]$ of the estimator Θ^*, dashed lines show the difference between the expected value and standard deviation $E[\Theta^*] - Var[\Theta^*]^{1/2}$ of the estimator Θ^*, dotted lines show several realizations of Θ^*